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Statistics of multiple sign changes in a discrete non-Markovian sequence

Satya N. Majumdar
Laboratoire de Physique Quantique (UMR C 5626 du CNRS), Université Paul Sabatier, 31062 Toulouse Cedex, France

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We study analytically the statistics of multiple sign changes in a discrete non-Markovian sequence $\psi_i=\phi_i+\phi_{i-1}$ $(i=1,2,\ldots,n)$ where ϕ_i's are independent and identically distributed random variables each drawn from a symmetric and continuous distribution $\rho(\phi)$. We show that the probability $P_m(n)$ of m sign changes up to n steps is universal, i.e., independent of the distribution $\rho(\phi)$. The mean and variance of the number of sign changes are computed exactly for all $n>0$. We show that the generating function $\tilde{P}(p,n)=\Sigma_{m=0}^{\infty}P_m(n)p^m \sim \exp[-\theta_d(p)n]$ for large n where the ''discrete'' partial survival exponent $\theta_d(p)$ is given by a nontrivial formula, $\theta_d(p)=\ln[\sin^{-1}(\sqrt{1-p^2})/\sqrt{1-p^2}]$ for $0\leqslant p\leqslant 1$. We also show that in the natural scaling limit $m\to\infty$, $n\to\infty$ but keeping $x=m/n$ fixed, $P_m(n)\sim\exp[-n\Phi(x)]$ where the large deviation function $\Phi(x)$ is computed. The implications of these results for Ising spin glasses are discussed.

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The probability $P_0(T)$ that a stochastic process $\psi(T)$ does not cross zero up to time T is a quantity of long-standing interest to both physicists and mathematicians [1,2] and has resurfaced recently with the new name ''persistence'' in the context of nonequilibrium systems [3]. A lot of recent efforts have been devoted to computing $P_0(T)$ for stationary Gaussian processes. Such a Gaussian stationary process (GSP) is completely specified by its two-point correlation function $C(T)=\langle\psi(0)\psi(T)\rangle$. For a wide class of correlation functions, it is known that $P_0(T)\sim\exp(-\theta T)$ for large T where the persistence exponent θ is usually nontrivial, depends on the full function $C(T)$, and is calculable exactly only in very few cases [3]. A natural generalization of $P_0(T)$ is $P_m(T)$, the probability of m zero crossings up to time T. The mean and the variance of the number of zero crossings up to time T of a GSP have been studied before [1]. For a smooth GSP where $C(T)=1-aT^2+\ldots$ for small T with $a>0$, the mean is given by Rice's formula [4], $\langle m\rangle/T=\sqrt{-C''(0)}/\pi$ and the variance by a more complicated formula due to Bendat [5]. Recently it was shown [6] that for a smooth GSP the generating function $\tilde{P}(p,T)=\Sigma_{m=0}^{\infty}P_m(T)p^m\sim\exp[-\theta(p)T]$ for large T where the ''partial survival'' exponent $\theta(p)$ varies smoothly from $\theta(0)=\theta$ to $\theta(1)=0$ as p varies continuously from 0 to 1. In Ref. [6], the exponent $\theta(p)$ was computed exactly for the one-dimensional Ginzburg-Landau model of deterministic coarsening and also approximately within the independent interval approximation for other smooth processes such as the diffusion equation. The only process for which a closed-form expression of $\theta(p)$ exists, so far, is the random acceleration problem [7,8] which corresponds to a GSP [3] with correlator $C(T)=[3\exp(-T/2)-\exp(-3T/2)]/2$. For this problem, the exponent $\theta(p)$ for $0\leqslant p\leqslant 1$ is given by the formula, $\theta(p)=\frac{1}{4}[1-6/\pi\sin^{-1}(p/2)]$.

In this Rapid Communication, we study the statistics of multiple crossings or sign changes in a discrete sequence $\psi_1,\psi_2,\ldots,\psi_n$ as opposed to a continuous process $\psi(T)$ discussed in the previous paragraph. This study is motivated by the recent works on the persistence of a discrete sequence [9–11]. The principal motivation for studying the persistence of a discrete sequence is twofold. First, in various experiments and numerical simulations to measure the persistence $P_0(T)$ of a continuous stochastic process $\psi(T)$, one usually samples the continuous process only at discrete time points separated by a fixed window size ΔT and checks whether the process has retained the same sign at all these discrete times. Some information gets lost due to this discretization since the continuous process $\psi(T)$ may have crossed and recrossed zero in between two successive discrete points. Thus, the ''discrete-time'' persistence $P_0(n)$ [i.e., the probability that the sequence $\psi(0)$, $\psi(\Delta T)$, $\psi(2\Delta T),\ldots,$ $\psi(n\Delta T=T)$ have the same sign] is usually greater than the continuous time persistence $P_0(T)$. In Ref. [9], it was shown that $P_0(n)\sim\exp[-\theta_d n]$ where the exponent θ_d depends continuously on the window size ΔT. The second motivation for studying the persistence of a sequence follows from the observation [10] that many processes in nature such as weather records are stationary under translations in time only by an integer multiple of a basic period (which can be chosen to be unity without loss of generality). It was shown in Ref. [10] that for a wide class of such processes, the continuous time persistence $P_0(T)$ is the same as the persistence $P_0(n)$ of the corresponding discrete sequence obtained from the measurement of the process only at integer times. A natural generalization of $P_0(n)$ is clearly $P_m(n)$, the probability that there are m sign changes along a sequence of size n.

The exact calculation of $P_m(n)$ for an arbitrary stationary sequence seems difficult. It is therefore important to find exactly solvable cases. In this paper we present exact results for $P_m(n)$ for a specific sequence which was introduced in Ref. [10],

$$\psi_i=\phi_i+\phi_{i-1},\quad i=1,2,\ldots,n \tag{1}$$

where ϕ_i's are independent and identically distributed (i.i.d) random variables, not necessarily Gaussian, each drawn from the same symmetric continuous distribution $\rho(\phi)$. The variables ψ_i's have only nearest-neighbor correlations. The sequence in Eq. (1) is stationary but non-Markovian since ψ_i depends not just only on ψ_{i-1} but on all the preceding members of the sequence [10]. This sequence

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appears as a limiting case of the diffusion equation on a hierarchical lattice [10]. It also appears in the one-dimensional Ising spin-glass problem where ψ_i represents the energy cost to flip the ith spin [12]. In Ref. [10], the persistence $P_0(n)$ for this sequence was computed exactly for all n and remarkably $P_0(n)$ was found to be *universal*, i.e., independent of the distribution $\rho(\phi)$. In particular, it was that $P_0(n) \sim \exp[-\theta_d n]$ for large n with $\theta_d = \ln[\pi/2]$. The persistence $P_0(n)$ was shown to be identical to the average fraction of metastable configurations (originally computed in Ref. [12]) in the corresponding Ising spin-glass chain [10].

The purpose of this Rapid Communication is to show that $P_m(n)$ for any $m \geq 0$ can also be calculated exactly for the sequence in Eq. (1) and turns out to be universal. Let us summarize our main results which are all independent of the distribution $\rho(\phi)$:

(i) We show that the mean number of sign changes upto n steps (i.e., when the sequence size is $n+1$) is given by the exact formula, $\langle m \rangle = n/3$ for all $n > 0$.

(ii) The variance is given by the formula, $\sigma_n^2 = \langle m^2 \rangle - \langle m \rangle^2 = [16n + 3 + \delta_{n,1}]/90$ for all $n > 0$.

(iii) We show that analogus to its continuous counterpart, the generating function $\tilde{P}(p,n) = \Sigma_{m=0}^{n} p^m P_m(n) \sim \exp[-\theta_d(p) n]$ where the "discrete partial survival" exponent $\theta_d(p)$ is given by the closed form expression

$$\theta_d(p) = \ln\left[\frac{\sin^{-1}(\sqrt{1-p^2})}{\sqrt{1-p^2}}\right], \quad 0 \leq p \leq 1. \tag{2}$$

This result can be analytically continued to $p \geq 1$.

(iv) We also show that in the limit $m \to \infty$, $n \to \infty$ but keeping $x = m/n$ fixed, $P_m(n) \sim \exp[-n\Phi(x)]$ where $\Phi(x)$ is a universal large deviation function that we compute.

We start by defining $P_{m,n}^{\pm}(\phi_0)$ to be the joint probability that the first member of the sequence in Eq. (1) $\pm\psi_1 > 0$ and that the sequence undergoes m sign changes up to n steps, given the value of ϕ_0. It is then easy to see that they satisfy the following recursion relations:

$$P_{m,n+1}^{+}(\phi_0) = \int_{-\phi_0}^{\infty} d\phi_1 \rho(\phi_1) F_{m,n}^{+}(\phi_1),$$

$$P_{m,n+1}^{-}(\phi_0) = \int_{-\infty}^{-\phi_0} d\phi_1 \rho(\phi_1) F_{m,n}^{-}(\phi_1), \tag{3}$$

where $F_{m,n}^{\pm}(\phi_1) = P_{m,n}^{\pm}(\phi_1) + P_{m-1,n}^{\mp}(\phi_1)$ and the initial conditions are $P_{m,0}^{\pm}(\phi_0) = 0$ for $m > 0$, $P_{0,0}^{+}(\phi_0) = \int_{-\phi_0}^{\infty} \rho(\phi_1) d\phi_1$, and $P_{0,0}^{-}(\phi_0) = 1 - P_{0,0}^{+}(\phi_0)$. The generating functions $\tilde{P}_n^{\pm}(p,\phi_0) = \Sigma_0^{\infty} P_{m,n}^{\pm}(\phi_0) p^m$ then satisfy the recursion relations

$$\tilde{P}_{n+1}^{+}(p,\phi_0) = \int_{-\phi_0}^{\infty} d\phi_1 \rho(\phi_1) \tilde{F}_n^{+}(p,\phi_1),$$

$$\tilde{P}_{n+1}^{-}(p,\phi_0) = \int_{-\infty}^{-\phi_0} d\phi_1 \rho(\phi_1) \tilde{F}_n^{-}(p,\phi_1), \tag{4}$$

where $\tilde{F}_n^{\pm}(p,\phi_1) = \tilde{P}_n^{\pm}(p,\phi_1) + p\tilde{P}_n^{\mp}(p,\phi_1)$ with the initial conditions, $\tilde{P}_0^{+}(p,\phi_0) = \int_0^{\infty} \rho(\phi_1) d\phi_1$ and $\tilde{P}_0^{-}(p,\phi_0) = 1 - \tilde{P}_0^{+}(p,\phi_0)$. Further simplification can be made by differenting Eq. (4) with respect to ϕ_0 followed by a change of variable $\phi_0 \to u = \int_0^{\phi_0} \rho(\phi) d\phi$ and then using the symmetry $\rho(\phi) = \rho(-\phi)$. Writing $\tilde{P}_n^{\pm}(p,\phi_0) = \tilde{p}_n^{\pm}(u)$ (suppressing the p dependence for convenience) we find two coupled nonlocal recursion relations

$$\frac{d\tilde{p}_{n+1}^{\pm}(u)}{du} = \pm[\tilde{p}_n^{\pm}(-u) + p\tilde{p}_n^{\mp}(-u)], \tag{5}$$

with the initial conditions $\tilde{p}_0^{\pm}(u) = 1/2 \pm u$ and the boundary conditions $\tilde{p}^{\pm}(\mp 1/2) = 0$. Note that the explicit dependence on the distribution $\rho(\phi)$ disappears in Eq. (5). As a result, all further quantities computed from these recursion relations will be independent of $\rho(\phi)$ provided $\rho(\phi)$ is symmetric and continuous.

In principle, one can solve the recursion relations in Eq. (4) by the generating function method. However to calculate the mean and the variance of zero crossings, it is simpler to directly analyze Eq. (5). Let $E_n^{\pm}(\phi_0) = \Sigma_{m=0}^{\infty} m P_{m,n}^{\pm}(\phi_0)$ be the expected number of sign changes up to n steps with the first member ψ_1 positive (or negative) and given ϕ_0. Let us write $E_n^{\pm}(\phi_0) = e_n^{\pm}(u)$ after making the change of variable $\phi_0 \to u$. The average number of crossings is then given by $\langle m \rangle = \int_{-\infty}^{\infty} [E_n^{+}(\phi_0) + E_n^{-}(\phi_0)] \rho(\phi_0) d\phi_0 = \int_{-1/2}^{1/2} [e_n^{+}(u) + e_n^{-}(u)] du$. Differentiating Eqs. (5) once with respect to p and putting $p = 1$, we get

$$\frac{de_{n+1}^{\pm}}{du} = \pm[e_n^{+}(-u) + e_n^{-}(-u)] + u \pm \frac{1}{2}, \tag{6}$$

with the initial conditions $e_0^{\pm}(u) = 0$ and the boundary conditions $e_n^{\pm}(\mp 1/2) = 0$. These recursion relations can be solved exactly and one gets $e_{n+1}^{\pm} = (1/2 \pm u)(n-4)/12 \pm u^3/3 + u^2/2 \pm u/2 + 1/6$. Hence, $e_n(u) = e_n^{+}(u) + e_n^{-}(u) = u^2 + n/3 - 1/12$. This then gives the exact result $\langle m \rangle = \int_{-1/2}^{1/2} e_n(u) du = n/3$ for all $n \geq 0$, independent of $\rho(\phi)$.

The variance $\sigma_n^2 = \langle m^2 \rangle - \langle m \rangle^2$ can be computed in a similar way by differentiating Eq. (5) twice with respect to p, putting $p = 1$, and solving the resulting recursion relations. The functions $G_n^{\pm}(\phi_0) = \Sigma_{m=0}^{\infty} m(m-1) P_{m,n}^{\pm}(\phi_0) = g_n^{\pm}(u)$ satisfy the following inhomogeneous recursion relations for $n > 0$:

$$\frac{dg_{n+1}^{\pm}}{du} = \pm[g_n^{+}(-u) + g_n^{-}(-u)] \pm 2e_n^{\mp}(-u), \tag{7}$$

with the initial conditions $g_0^{\pm}(u)=g_1^{\pm}(u)=0$ and boundary conditions $g_n^{\pm}(\mp 1/2)=0$. Using the known values of $e_n^{\pm}(u)$ one can again solve Eq. (7) explicitly. This finally gives $\langle m(m-1)\rangle=\int_{-1/2}^{1/2}[g_n^{+}(u)+g_n^{-}(u)]du=(10n^2-14n+3)/90$ for all $n\geqslant 2$. Using $\langle m\rangle=n/3$, one gets $\sigma_n^2=[16n+3+\delta_{n,1}]/90$ for all $n>0$, again independent of $\rho(\phi)$.

We now turn to the calculation of the partial survival exponent. We expect that for large n, $\tilde{p}_n^{\pm}(u)\approx\lambda^{-n}f^{\pm}(u)$ where $\lambda=\exp[\theta_d(p)]$. Substituting this asymptotic form in Eq. (5) we get the nonlocal eigenvalue equation

$$\frac{df^{\pm}(u)}{du}=\pm\lambda[f^{+}(-u)+f^{-}(-u)], \tag{8}$$

subject to the two boundary conditions, $f^{+}(-1/2)=0$ and $f^{-}(1/2)=0$. Diagonalizing Eq. (8) and solving the resulting nonlocal equations we get the most general solutions of Eq. (8),

$$f^{\pm}(u)=A^{\pm}\cos(\mu u)+B^{\pm}\sin(\mu u), \tag{9}$$

where $\mu=\lambda\sqrt{1-p^2}$ and the four constants $A^{\pm}$ and $B^{\pm}$ can be written in terms of only two unknown constants a and b via the relations, $A^{+}=ap$, $A^{-}=b\sqrt{1-p^2}-a$, $B^{+}=bp$, and $B^{-}=a\sqrt{1-p^2}-b$. The solution in Eq. (9) must satisfy the two boundary conditions $f^{\pm}(\mp 1/2)=0$ which give two homogeneous linear equations for the unknown constants a and b. Eliminating a and b from these two equations one gets $\mu=\sin^{-1}[\sqrt{1-p^2}]$ and hence $\lambda=\sin^{-1}[\sqrt{1-p^2}]/\sqrt{1-p^2}$. Using the relation $\theta_d(p)=\ln\lambda$, we obtain the result in Eq. (2), once again independent of $\rho(\phi)$. Note that for $p=0$, $\theta_d(p)$ reduces to the usual discrete persistence exponent $\theta_d=\ln(\pi/2)$.

The expression for $\theta_d(p)$ in Eq. (2) is valid in the range $0\leqslant p\leqslant 1$. However, in principle, one can define the generating function $\tilde{P}(p,n)=\Sigma_{m=0}^{\infty}P_m(n)p^n$ even for $p>1$. Then for $p>1$ one expects $\tilde{P}(p,n)$ to diverge as $n\to\infty$ indicating $\theta_d(p)$ becomes negative for $p>1$. Indeed one can easily get the result for $p>1$ by analytically continuing the expression in Eq. (2) to the range $p\geqslant 1$ and this gives

$$\theta_d(p)=\ln\left[\frac{\ln(p+\sqrt{p^2-1})}{\sqrt{p^2-1}}\right]. \tag{10}$$

Thus $\theta_d(p)$ tends to $-\infty$ rather slowly as $\theta_d(p)\sim\ln[\ln(2p)/p]$ as $p\to\infty$.

We next analyze the distribution $P_m(n)$ in an interesting scaling limit. Since the average number of crossings scale linearly with the size n as $\langle m\rangle=n/3$, a natural scaling limit is when $m\to\infty$, $n\to\infty$ but keeping the ratio $x=m/n$ fixed. In this limit, we show that $P_m(n)\sim\exp[-n\Phi(x)]$ where $\Phi(x)$ is a large deviation function which is universal, i.e., independent of the distribution $\rho(\phi)$. Large deviation functions associated with different physical observables have appeared before in the context of various nonequilibrium systems [13]. For example, the large deviation probabilities associated with the occupation time in discrete renewal type processes have been computed recently [14]. The present model provides an example where there is a large deviation function associated with the number of zero crossings that can be computed explicitly. Indeed, substituting the ansatz $P_m(n)\sim\exp[-n\Phi(x)]$ in the generating function one gets $\tilde{P}(p,n)\sim\Sigma_{m=0}^{\infty}P_m(n)p^m\sim\int_0^{\infty}dx\exp[-n\{\Phi(x)-x\log p\}]$. In the large n limit the integral can be evaluated by the steepest descent method and one gets $\tilde{P}(p,n)\sim\exp[-nG(p)]$ where $G(p)=\min_x\{\Phi(x)-x\log p\}$. On the other hand, by definition, $\tilde{P}(p,n)\sim\exp[-\theta_d(p)n]$ for large n. This establishes the relation, $\min_x\{\Phi(x)-x\log p\}=\theta_d(p)$. Thus $\theta_d(p)$ is just the Legendre transform of $\Phi(x)$. Inverting this Legendre transform we get

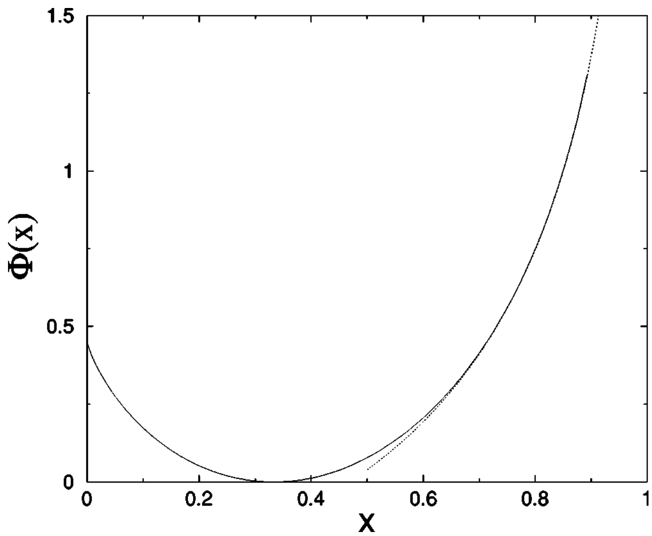

FIG. 1. The large deviation function $\Phi(x)$ plotted against x. The solid line represents the function obtained using Mathematica. The dotted line represents the analytical asymptotic form $\Phi(x)=-\ln(1-x)-1+(1-x)\ln 2$ in the limit $x\to 1$. The function $\Phi(x)\to\ln(\pi/2)=0.451\,583\ldots$ as $x\to 0$.

$$\Phi(x)=\max_p[x\log p+\theta_d(p)], \tag{11}$$

where $\theta_d(p)$ is given exactly by Eqs. (2) and (10) in the range $p\geqslant 0$. Note that determining $\Phi(x)$ from Eq. (11) requires a knowledge of $\theta_d(p)$ not just in the range $0\leqslant p\leqslant 1$ but also for $p\geqslant 1$. Thus we will need both the formulas in Eqs. (2) and (10).

We have obtained $\Phi(x)$ from Eq. (11) using mathematica and is displayed in Fig. 1 Since the number of crossings $m\leqslant n$, the allowed range of x is $0\leqslant x\leqslant 1$. One can analytically determine the behavior of $\Phi(x)$ in the three limits $x\to 0$, $x\to 1$ and $x\to 1/3$. First consider the limit $x\to 0$. This corresponds to $p\to 0$ limit of $\theta_d(p)$. Expanding Eq. (2) for small p, we get $\theta_d(p)\approx\ln(\pi/2-p)$. Substituting this in Eq. (11) and maximizing with respect to p gives $\Phi(x)\approx\ln(\pi/2)+x\log(x)$ as $x\to 0$. Next consider the opposite limit when $x\to 1$. This limit correspond to $\theta_d(p)$ in the limit $p\to\infty$. Hence, we need to now use the analytically continued formula in Eq. (10). Expanding Eq. (10) for large p, we get $\theta_d(p)\approx\ln[\ln(2p)/p]$ to leading order. Substituting this asymptotic form in Eq. (11) and maximizing with respect to

p we get $\Phi(x)\approx -\ln(1-x)-1+(1-x)\ln 2$ as $x\to 1$. In Fig. 1, this asymptotic form is shown by the dotted line to which $\Phi(x)$ approaches rather quickly as $x\to 1$.

The most interesting limit, however, is when $x\to 1/3$, i.e. $m\to\langle m\rangle$. This limit in x corresponds to $p\to 1$ limit of $\theta_d(p)$. It is easy to see that both the limits $p\to 1^-$ and $p\to 1^+$ yield the same result. Let us consider the case when $p=1-\epsilon$ where $\epsilon\to 0$. Expanding Eq. (2) in powers of ϵ, we get $\theta_d(p)\approx \epsilon/3+7\epsilon^2/90+O(\epsilon^3)$. Substituting this in Eq. (11) and maximizing with respect to $p=1-\epsilon$, we get as $x\to 1/3$,

$$\Phi(x)\approx \frac{45}{16}\left(x-\frac{1}{3}\right)^2. \tag{12}$$

This limiting form can also be derived independently from a central limit theorem. To see this we write the number of sign changes m as the sum $m=\Sigma_{i=1}^{n} w_i$ with $w_i=1-\theta(\psi_i\psi_{i+1})$ and $\theta(x)$ is the Heaviside step function. Thus, $m-\langle m\rangle=\Sigma_{i=1}^{n}(w_i-\langle w_i\rangle)$. Clearly in the limit $m\to\langle m\rangle$, the variables $(w_i-\langle w_i\rangle)$ become only weakly correlated. Then in the limit when n is much larger than the correlation length between these variables one expects the central limit theorem to hold predicting a Gaussian distribution for m, $P_m(n)\sim\exp[-(m-\langle m\rangle)^2/2\sigma_n^2]$. Using the already derived results $\langle m\rangle=n/3$ and $\sigma_n^2\approx 8n/45$ for large n, we find $P_m(n)\sim\exp[-45n(x-1/3)^2/16]$ thus yielding the same $\Phi(x)$ as in Eq. (12). Thus this limit provides an indipendent check of our results for the mean and the variance. The three limiting behaviors of $\Phi(x)$ are summmarized as follows:

$$\Phi(x)\approx\begin{cases}\ln(\pi/2)+x\log x, & x\to 0,\\ \frac{45}{16}\left(x-\frac{1}{3}\right)^2, & x\to 1/3,\\ -\ln(1-x)-1+(1-x)\ln 2, & x\to 1.\end{cases} \tag{13}$$

We conclude with a discussion of the implications of our results for an Ising spin-glass chain described by the Hamiltonian, $H=-\Sigma J_{i,i+1}s_is_{i+1}$ with $s_i=\pm 1$ and the bonds $J_{i,i+1}$'s are i.i.d random variables each drawn from the same symmetric and continuous distribution. A spin will be called metastable if the cost of energy to flip it under zero-temperature Glauber dynamics is positive, i.e., $\Delta E_i=2s_i[J_{i-1,i}s_{i-1}+J_{i,i+1}s_{i+1}]>0$. A given spin configuration (with fixed J's) consists of alternate domains of metastable and nonmetastable spins. A natural question is what is the average (over disorder) probability $P(m,n)$ that there are m such domains in a chain of length n. Defining the variables $\phi_i=2J_{i,i+1}s_is_{i+1}$ which are also i.i.d. variables, we see that the energy costs $\Delta E_i=\phi_i+\phi_{i-1}$ form exactly the sequence studied in this Rapid Communication. The average domain number probability $P(m,n)$ is then identical to the probability of having $2m$ sign changes in the sequence $\{\Delta E_i\}$ up to n steps, i.e., $P_{2m}(n)$ that has been computed exactly in this paper. Clearly for $m=0$, $P(0,n)=P_0(n)$ is just the fraction of fully metastable configurations (out of the 2^n configurations) at zero temperature and is the same as the persistence of the sequence $\{\Delta E_i\}$ [10]. Note that one can easily generalize this question of the domain number probability to higher dimensions as well. The study of the statistics of the domains of metsatable spins in higher-dimensional spin glasses may provide interesting insights into the nature of the low-temperature phase.

A natural extension of the present work would be to compute the statistics of crossings of a sequence with larger memory, $\psi_i=\Sigma_{j=0}^{k}\phi_{i-j}$ where $k\geqslant 1$ and ϕ_j's are i.i.d random variables. Exact computation of the persistence as well as the large deviation function associated with multiple crossings for $k>1$ remains a challenging open problem.

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